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Neutrino Oscillation based on the Mixings with a Heavy Right-Handed Neutrino

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Abstract

We study a consistent explanation for both deficits of solar neutrino and atmospheric neutrino due to the neutrino oscillation induced by the mixing of light neutrinos with a heavy right-handed neutrino. We propose such a phenomenological neutrino mass matrix that realizes this scenario in the simple three generation left- and right- handed neutrino framework. Although this model contains only one nonzero mass eigenvalue ~ 10 eV for these light neutrino states at the first approximation level, it can be expected to explain consistently both deficiency due to the appropriate higher order corrections. A suitable hot dark matter candidate is naturally included in it as its own feature.

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It is one of the most interesting issues to clarify the neutrino mass problem in the present particle physics. It has the great influences not only to the consideration of new physics beyond the standard model but also to the study of astrophysics. Now we have some clues for this problem. The deficiency of solar neutrinos[1] and atmospheric neutrinos[2] have been shown to be explained by $\nu_e \rightarrow \nu_x$ and $\nu_\mu \rightarrow \nu_y$ oscillation, respectively. The predicted neutrino masses and mixings from these observations are followings. For solar neutrino problem[3],[1]

(i) MSW small mixing solution:

$$\begin{aligned} &\Delta m_{xe} \equiv (m^2_{\nu_x} - m^2_{\nu_e}) \sim (0.3 - 1.2) \times 10^{-5}\mathrm{eV}^2, \\ &\sin^2 2\theta \sim (0.4 - 1.5) \times 10^{-2}, \end{aligned} \tag{1}$$

(ii) MSW large mixing solution:

$$\begin{aligned} &\Delta m_{xe} \equiv (m^2_{\nu_x} - m^2_{\nu_e}) \sim (0.3 - 3) \times 10^{-5}\mathrm{eV}^2, \\ &\sin^2 2\theta \sim 0.6 - 0.9, \end{aligned} \tag{2}$$

and for atmospheric neutrino problem[4],

$$\begin{aligned} &\Delta m_{y\mu} \equiv |m^2_{\nu_y} - m^2_{\nu_\mu}| \sim (0.5 - 50) \times 10^{-2}\mathrm{eV}^2, \\ &\sin^2 2\theta \gtrsim 0.5. \end{aligned} \tag{3}$$

Usually it is considered that ν_x is ν_μ and ν_y is ν_τ. It is a very interesting subject to realize these parameters in the suitable particle physics model. There are various studies of the neutrino mass matrix which can explain these values consistently[5, 6].

On the other hand, it is well known now that the most amount of mass in the universe must be stored in the nonbaryonic dark matter. There have been proposed two possible candidates of dark matter, that is, the hot dark matter and the cold dark matter. However, the analyses based on the data on the structure of the universe which are now available on a wide range of distance scales have shown that these dark matter models fail to explain the structures on small or large distance scales, respectively. It was suggested recently that a cold + hot dark matter model agrees well with astrophysical observations if there is one neutrino species with ~ 5 eV mass[7, 8]. From this cosmological point of view, it

[1] There is another vacuum oscillation solution. The oscillation parameters for this solution are $\Delta m^2 \sim (0.5 - 1.1) \times 10^{-10}\mathrm{eV}^2$, $\sin^2 2\theta \sim 0.8 - 1.0$. However, we do not consider this solution in this paper.

is a well motivated subject to introduce ∼10 eV neutrino as a candidate of dark matter into the particle physics model in the natural way. In fact, there are many trials of the model building to predict these neutrino masses[9]. However, it seems not to be so easy to construct models which accomodates these features naturally.

Usually either the seesaw mechanism[10] or the loop effects are used to realize the sufficiently small neutrino masses. In the ordinary seesaw mechanism, three right-handed neutrinos get the large Majorana masses and the hierarchy $m_{\nu_\tau} \gg m_{\nu_\mu} \gg m_{\nu_e}$ appears. However, in such a scheme it is rather difficult to explain the above mentioned observations, simultaneously[5, 6]. On the other hand, if we only use the loop effects, the model often becomes complex and it is necessary to introduce various exotic fields, that is, extra color triplets, doubly charged fields and so on[11]. Under this situation it will be worthy to examine alternative possibility for the neutrino mass matrix which may explain the small neutrino masses and mixings appropriate for the above mentioned neutrino problems.

In this paper we propose a scenario which can accommodate the above mentioned features and also can be embedded into the framework similar to the standard model with three generation left- and right-handed neutrinos. Our strategy is the following. Firstly we require the realization of the suitable mixings and the dark matter candidate mass and after that we introduce the appropriate mass differences without violating the feature of above mixings. In this scenario only one right-handed neutrino N_R has a large Majorana mass and remaining light neutrinos have small mixings with it. Although there is only one nonzero mass eigenvalue among these light neutrinos at this first approximation stage due to the seesaw mechanism[10], the required neutrino oscillations can occur if suitable mass perturbations expected from these mixings are introduced.

To discribe the basic feature of this scenario, we consider the model defined by the following effective mass terms in the first approximation,

$$-\mathcal{L}_{\rm mass} = \sum_{\alpha=1}^{4} m_\alpha \psi_\alpha N_R + \frac{1}{2} M N_R N_R + h.c., \tag{4}$$

where ψ_α represents the charge conjugate states of ordinary left-handed neutrinos $\psi_{Li}(i = 1 \sim 3)$ and also a right-handed sterile one ψ_R. These and N_R are the weak interaction eigenstates and $m_\alpha \ll M$ is assumed. This can be straightforwardly embedded into the full three generation neutrino model without any large changes in the following results. The rank of a 5×5 mass matrix derived from this $\mathcal{L}_{\rm mass}$ is two. One nonzero eigenvalue is

large enough compared with the other one. Using seesaw mechanism we can resolve the mixing between a heavy state and the remaining four light states. Under such a basis the mass matrix of four light states are written as

$$M_{\rm light} = M \begin{pmatrix} \mu_1^2 & \mu_1\mu_2 & \mu_1\mu_3 & \mu_1\mu_4 \\ \mu_1\mu_2 & \mu_2^2 & \mu_2\mu_3 & \mu_2\mu_4 \\ \mu_1\mu_3 & \mu_2\mu_3 & \mu_3^2 & \mu_3\mu_4 \\ \mu_1\mu_4 & \mu_2\mu_4 & \mu_3\mu_4 & \mu_4^2 \end{pmatrix}, \tag{5}$$

where $\mu_\alpha = m_\alpha/M (\ll 1)$. As is easily checked, $M_{\rm light}$ is diagonalized as $U^{(\nu)} M_{\rm light} U^{(\nu)T}$ by using the matrix

$$U^{(\nu)} = \begin{pmatrix} \frac{\mu_2}{\xi_1} & -\frac{\mu_1}{\xi_1} & 0 & 0 \\ \frac{\mu_1\mu_3}{\xi_1\xi_2} & \frac{\mu_2\mu_3}{\xi_1\xi_2} & -\frac{\xi_1}{\xi_2} & 0 \\ \frac{\mu_1\mu_4}{\xi_2\xi_3} & \frac{\mu_2\mu_4}{\xi_2\xi_3} & \frac{\mu_3\mu_4}{\xi_2\xi_3} & -\frac{\xi_2}{\xi_3} \\ \frac{\mu_1}{\xi_4} & \frac{\mu_2}{\xi_4} & \frac{\mu_3}{\xi_4} & \frac{\mu_4}{\xi_4} \end{pmatrix}, \tag{6}$$

where $\xi_n^2 = \sum_{\alpha=1}^{n+1} \mu_\alpha^2$. At this stage one of the mass eigenvalues is $M\mu_4^2$ and others are zero and then the constraints for the squared mass differences (1) $\sim$ (3) cannot be satisfied. In order to prepare the suitable mass differences among these light neutrinos without violationg the mixing property shown above, we will introduce a mass perturbation,

$$M_{\rm per} = U^{(\nu)T} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_1 & 0 & 0 \\ 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} U^{(\nu)}. \tag{7}$$

The eigenvalues of the perturbed mass matrix $M_{\rm light} + M_{\rm per}$ are $0, M_1, M_2$ and $M\mu_4^2$.

Now we consider the oscillation phenomena in these four light states. The mass eigenstates ϕ_α is related to the weak interaction eigenstates ψ_α by the KM-mixing matrix $V^{(l)}$ for leptons as

$$\phi_\alpha = \sum_{\beta=1}^{4} V^{(l)}_{\alpha\beta}\psi_\beta, \tag{8}$$

where $V^{(l)}$ can be written as $V^{(l)} = U^{(\nu)}U^{(l)\dagger}$ by using the diagonalization matrix $U^{(l)}$ of the charged lepton mass matrix. Here we consider the basis on which both of the charged lepton mass matrix and the lepton charged current are diagonal. Using this mixing matrix,

we can write down the time evolution equation of these states,

$$i\frac{d}{dt}\psi_\alpha = \sum_{\beta=1}^{4} H_{\alpha\beta}\psi_\beta. \tag{9}$$

If we assume as usual that these states are ultra relativistic, Hamiltonian H is expressed as

$$H_{\alpha\beta} = \sum_{\gamma=1}^{4} V^{(l)\dagger}_{\alpha\gamma}\frac{\tilde{m}_\gamma^2}{2E}V^{(l)}_{\gamma\beta} + a_\alpha\delta_{\alpha\beta}, \tag{10}$$

where $\tilde{m}_\gamma$ is the γ-th mass eigenvalue of $M_{\rm light} + M_{\rm per}$. E is the energy of neutrinos. The potentials induced effectively through the weak interactions with matter are introduced as a_α, which should be taken as zero for the sterile neutrinos. In order to explain the neutrino oscillations and the dark matter mass introduced in the first part, we need to identify these four light states with the physical ones in our neutrino model. In the following discussion we will focus our attention on the case,

$$\psi_4 \rightarrow \text{a right-handed sterile neutrino}(\nu_s), \quad \psi_1, \psi_2, \psi_3 \rightarrow \nu_{eL}, \nu_{\mu L}, \nu_{\tau L}.$$

Under this identification to realize the suitable mixing angles for the solar and atmospheric neutrino problems and also the mass of the dark matter candidate simultaneously, it is necessary to require the following relations for each solution (i) and (ii),[2]

$$\begin{array}{llll} \text{(i)} & 16 \lesssim \frac{\mu_2}{\mu_1} \lesssim 32, & 1 \lesssim \frac{\mu_3}{\mu_2} \lesssim 2.4, & \frac{\mu_4}{\mu_3} \sim 10^p, \\ \text{(ii)} & 1.4 \lesssim \frac{\mu_2}{\mu_1} \lesssim 2.1, & 1 \lesssim \frac{\mu_3}{\mu_2} \lesssim 3, & \frac{\mu_4}{\mu_3} \sim 10^p, \end{array} \tag{11}$$

and the suitable mass as the dark matter candidate,

$$M\mu_4^2 \sim 10 \text{ eV}, \tag{12}$$

for the largest mass eigenvalue.[3] At this stage a positive constant p is a free parameter but it crucially depends on the BBN constraint[12]. Both values of p and $M\mu_4^2$ will be changed if the state identification for the dark matter candidate is altered. On the other hand, since the introduction of $M_{\rm per}$ does not modify the mixing property (6), the constraints for the mass differences are directly imposed on M_1 and M_2. That is, if we assume $M_1 \sim 10^{-3}$ eV

[2]Here we assumed $V^{(l)} = U^{(\nu)}$. It should be also noted that $\mu_2 \sim \mu_3 \ll \mu_4$ is necessary to satisy the constraints (1)$\sim$(3) for mixings because of the form of mixing matrix (6).

[3]This value should be understood as the roughly required order of maginitude. More detailed analysis will be presented in the later part.

and $M_2 \sim 10^{-1}$ eV, the required values for the squared mass differences in the solar and atmospheric neutrino deficits can be built in the mass matrix $M_{\rm light} + M_{\rm per}$. For this mass matrix with the parameter setting (11), in the (ψ_2, ψ_3) sector the mixing angle becomes very large as $\sin^2 2\theta \sim 1$ and the effective squared mass difference is M_2^2. For these values, the atmospheric ν_μ deficit can be explained by the $\psi_2 \leftrightarrow \psi_3$ oscillation. In the (ψ_1, ψ_2) sector, the squared mass difference is M_1^2, which may realize the appropriate values (1) and (2) for the MSW solution[13] of solar neutrino problem. As to the mixing angle, both of the small mixing and large mixing solution are possible depending on the ratio of μ_1 and μ_2. Moreover, it is interesting that in the present scenario only nonzero mass eigenvalue $M\mu_4^2$ in the first approximation can be set just in the appropriate region for the hot dark matter, which can explain the structure formation of the universe. The mass matrix $M_{\rm light} + M_{\rm per}$ is found to satisfy all preferable features for the neutrino problems.

It will be useful here to present some comments related to the observation. In this scheme the solar neutrino deficit is explained by $\nu_e \to \nu_s$[14] and the atmospheric neutrino deficit is due to the ν_μ-ν_τ oscillation. The possibility of $\nu_e \to \nu_s$ will be examined by the future solar neutrino experiments in Super-Kamiokande and SNO as pointed out in [15]. Also the observation of ν_μ-ν_τ oscillation will be helpful to distinguish this scenario from other possibilities.

It will be also interesting to consider in what kind of supersymmetric models this type of mass matrix can be realized. To see this point in more detailed, we concretely write down eq.(7) in the case of $\mu_1 \ll \mu_2 \simeq \mu_3 \ll \mu_4$,

$$M_{\rm per} \simeq \left(\begin{array}{cccc} A\mu_1^2 & A\mu_1\mu_2 & B\mu_1\mu_3 & -C\mu_1\mu_4 \\ A\mu_1\mu_2 & A\mu_2^2 & B\mu_2\mu_3 & -C\mu_2\mu_4 \\ B\mu_1\mu_3 & B\mu_2\mu_3 & A\mu_3^2 & -C\mu_3\mu_4 \\ -C\mu_1\mu_4 & -C\mu_2\mu_4 & -C\mu_3\mu_4 & 2C\mu_3^2 \end{array} \right), \qquad (13)$$

where

$$A \sim \frac{1}{2}(M_1 + M_2)\mu_3^{-2}, \quad B \sim \frac{1}{2}(-M_1 + M_2)\mu_3^{-2}, \quad C \sim M_2\mu_4^{-2}.$$

Here we should note that in eq.(13) there appears the analogous structure for μ_α to eq.(5). In eq.(5) μ_α stands for an effective mixing between ψ_α and N_R. If $M_{\rm per}$ is induced as the loop effects through the suitable Yukawa couplings, the factors $\mu_\alpha\mu_\beta$ in eq.(13) can be interpreted just as the ratio of the product of corresponding Yukawa couplings which compose the loop diagrams. This feature at least suggests that compared with the MSSM

contents, the necessity of the extension of doublet Higgs sector and also the introduction of the singlet sector which has the couplings with other fields similar to the ones of a heavy right-handed neutrino N_R.[4] Related to the model construction, we can settle the parameters in eq.(4) to satisfy eqs.(11) and (12) in the case of $M \sim 10^{12+2q}$ GeV,

$$\begin{aligned} &\text{(i)} \qquad m_4 \sim 10^{2+q}, \quad m_3 \sim m_2 \sim 10^{2+q-p}, \qquad m_1 \sim 5 \times 10^{q-p}, \\ &\text{(ii)} \qquad m_4 \sim 10^{2+q}, \quad m_3 \sim m_2 \sim m_1 \sim 10^{2+q-p}, \end{aligned} \tag{14}$$

where these values are written in the GeV unit. A constant q is a free parameter here but in principle it will be determined by fixing the generating mechanism of a heavy right-handed neutrino mass M. For the concrete model construction it may be helpful to note the following points related to the parameter q. $M\mu_2^2 \sim M\mu_3^2 \sim 10^{1-2p}$eV is always satisfied independently of the value of q and also the value $m_4 \sim 1$ GeV corresponds to $M \sim 10^8$ GeV($q = -2$).

As is easily seen from the formulus (6), under the above parameter settings (11) or (14), the mixings between (ψ_1, ψ_2, ψ_3) and ψ_4 are crucially dependent on the parameter p. The osillations between them are very important from various viewpoints. Relating to this point, we should add here some discussions about the probability that the sterile neutrino becomes the dark matter candidate without conflicting with the BBN bound[15, 17]. We consider the neutrinos ν_L and ν_R whose mass terms are given by

$$-\mathcal{L}_{\rm mass} = m_D \bar{\nu}_L \nu_R + \frac{1}{2} m_R \nu_R \nu_R + h.c., \tag{15}$$

where we assume $m_D \ll m_R$ and then the light neutrino mass is given as $m_\nu = m_D^2/m_R$. Following the analysis in ref.[17], the relation between the distribution functions f_s, f_ν of sterile and active neutrinos at the period of nucleosynthesis is given by

$$\frac{f_s}{f_\nu} = \frac{6.0}{g_*^{\frac{1}{2}}} \left(\frac{m_D}{1 \text{ eV}}\right)^2 \left(\frac{1 \text{ keV}}{m_R}\right), \tag{16}$$

where g_* is the number of effective massless degrees of freedom at that time. In order to estimate m_D and m_R consistent with the BBN we take account of the relations

$$\frac{\Omega_s}{\Omega_\nu} = \frac{m_R f_s}{m_\nu f_\nu}, \qquad \frac{m_\nu}{\Omega_\nu} \simeq 92 h^2 \text{ eV} \tag{17}$$

[4]The trial in this direction is presented in ref.[16].

where $h = H_0/(100\text{km sec}^{-1}\ \text{MPc}^{-1})$ and H_0 is Hubble constant today. The ratio of energy density of particle species a to the critical energy density of the universe is represented as $\Omega_a \equiv \rho_a/\rho_c$. From eqs.(16) and (17) we get

$$m_D \sim 1.2 \times 10^{-1} g_*^{\frac{1}{4}} \Omega_s^{\frac{1}{2}} h. \qquad (18)$$

Following BBN scenario, the contribution of sterile neutrinos to the energy density at the time of primodial nucleosynthesis must be smaller than the contribution of a light neutrino species : $f_s \lesssim 0.4 f_\nu$. From this fact, using (17) we get

$$m_R \gtrsim 230 h^2 \Omega_s \text{ eV}. \qquad (19)$$

In case of $m_R \gg m_D$, the mixing angle θ is given by $\theta \sim m_D/m_R$ and then

$$m_R^2 \sin^2\theta \sim m_D^2 \sim 1.5 \times 10^{-2} h^2 g_*^{\frac{1}{2}} \Omega_s \text{ eV}^2. \qquad (20)$$

If we take $g_* \sim 10.8$, $h \sim 0.5$ and $\Omega_s \sim 0.3$ as the typical values, we can roughly estimate m_R and $m_R^2 \sin^2\theta$ as $m_R \gtrsim 17$ eV and $m_R^2 \sin^2\theta \sim 10^{-2}$ eV^2.

This result can be applyed to (ψ_3, ψ_4) sector of the present model when the sterile neutrino ψ_4 is regarded as the dark matter candidate.[5] In fact, focussing on (ψ_3, ψ_4) sector in $M_{\text{light}} + M_{\text{per}}$ and noticing that $M\mu_4^2$ and $M\mu_3\mu_4$ can be corresponded to m_R and $m_R \sin\theta$ in the above arguments, we find some interesting features on the basis of the above BBN constraints. At first, if we take $p \sim 2$ under such constraints, we can set a value of M_2 freely in the range which is appropriate for the explanation of atmospheric neutrino problem independently of the mass of the dark matter candidate ψ_4. This is because the present mass perturbation M_{per} is such one that does not disturb the mixing property (6). On the other hand, from eq.(19) the sterile neutrino mass should be $\gtrsim 17$ eV as suggested in [15, 17], which is somehow larger value than one of the usual hot dark matter[7, 8].[6]

Some remarks related to this scenario are ordered. Firstly we should comment on the possibility to embed the present scenario into some underlying theories. We have already presented some statements on this point. Such models will not be the usual grand unified

[5]In the present model (ψ_2, ψ_4) sector has also the similar mixing to (ψ_3, ψ_4) sector. If we consider its effect, the mass bound for the sterile neutrino will become larger.

[6] If ν_τ is considered to be a dark matter candidate, such a constraint will not appear.

models with the ordinary seesaw mechanism, in which the right handed neutrinos are required to be all heavy. One promising possibility is a superstring inspired E_6 model, in which the group theoretical constraints on the Yukawa couplings become very weak. Usually it is not so easy to accomodate small neutrino masses and induce neutrino oscillation without bringing other phenomenological difficulty in that framework[21]. However, if we introduce unconventional field assignments[22] under suitable conditions in that model, it is possible to show that the similar mass matrix structure discussed here can be realized. The related study of this subject will be presented elsewhere[16].

Next it may be useful to present a brief discussion on other possibility of the state identification such as $\psi_2 \rightarrow$ a right-handed sterile neutrino(ν_s) and $\psi_1, \psi_3, \psi_4 \rightarrow \nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$. In our scheme the light right-handed neutrino has the mixing with left-handed active neutrinos. The BBN again severely constrains the mixing angle θ and the squared mass difference $\Delta\tilde{m}^2$ for a sterile neutrino mixing with left-handed active neutrinos ν[15, 18],

$$\begin{aligned} &\Delta\tilde{m}^2 \sin^4 2\theta \lesssim 5 \times 10^{-6}\ \mathrm{eV}^2 \quad (\nu = \nu_e), \\ &\Delta\tilde{m}^2 \sin^4 2\theta \lesssim 3 \times 10^{-6}\ \mathrm{eV}^2 \quad (\nu = \nu_{\mu,\tau}). \end{aligned} \tag{21}$$

These constraints rule out the large mixing MSW solution of solar neutrino problem due to $\nu_e \rightarrow \nu_s$ and also the explanation of atmospheric neutrino problem by $\nu_\mu \rightarrow \nu_s$. These constraints are derived under the assumption that the relic neutrino asymmetry L_ν is very small. Although it has been recently suggested in ref.[19] that for $L_\nu > 7 \times 10^{-5}$ both of these solutions can be consistent with BBN constraints, this seems to be unlikely realized. The scenario studied here seems to be an only allowed one following the consistency with the BBN.

We consider here only a diagonal charged lepton mass matrix. For the non-diagonal one like the Fritzsch mass matrix[20] the situation becomes more complicated and the introduction of complex phases will be necessary to make our scenario available as suggested in ref.[6].

In summary we proposed a neutrino oscillation scenario based on a suitable neutrino mass martix which can explain successfully both of the deficits of solar neutrino and atmospheric neutrino within the framework of three generation left and right handed neutrinos. Although in this scenario the suitable mixings and only one nonzero mass eigenvalue are realized in the first approximation level, the appropriate effective squared mass differences

for the explanation of both problems can be induced through the mass perturbation which does not violate the first approximated state mixings induced through the seesaw mechanism. An interesting feature of this scenario is that it can naturally contain a neutrino as the dark matter candidate with the appropriate mass.

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References

[1] R. Davis, Jr. *et al.*, in Proc. of the 21st International Cosmic Ray Conference, Adelaide, Australia, 1989, ed. R. J. Protheroe (University of Adelaide Press, Adelaide,1990), Vol.12, p.143; R. Davis, Prog. Part. Nucl. Phys. 32(1994), in press;

K. S. Hirata *et al.*, Phys. Rev. Lett. **65**(1990)1297; Phys. Rev. Lett. **66**(1990)9;

GALLEX collab., Phys. Lett. **B285**(1992)376; Phys. **B285**(1992)390;

SAGE collab., Phys. Rev. Lett. **67**(1991)3332; Phys. Lett. **B328**(1994)234.

[2] K. S. Hirata *et al.*, Phys. Lett. **B205**(1988)416; Phys. Lett. **B280**(1992)146; Y. Fukuda *et al.*, Phys. Lett. **B335**(1994)237;

D. Casper *et al.*, Phys. Rev. Lett. **66**(1991)2561; R. Becker-Szendy *et al.*, Phys. Rev. **D46**(1992)3720;

P. J. Litchfield, in International Europhysics Conference on High Energy Physics, Marseille, France, 1993.

[3] N. Hata, S. Bludman and P. Langacker, Phys. Rev. **D49** (1994)3622;

P. I. Krastev and A. Yu. Smirnov, Phys. lett. **B338**(1994)282.

[4] T. K. Gaisser, in TAUP93, Nucl. Phys. (Proc. Suppl.) **B35**(1994)209.

[5] K. S. Babu and Q. Shafi, Phys. Lett. **B294**(1992)235;

C. H. Albright, Z. Phys. **C56**(1992)587.

[6] M. Fukugita, T. Yanagida and M. Tanimoto, Prog. Theor. Phys. **89**(1993)263;

K. Matumoto and D. Suematsu, Mod. Phys. Lett. **A9**(1994)41.

[7] E. L. Wright *et al.*, Astrophys. J. **396**(1992)L13;

M. Davis, F. J. Summers and D. Schlegal, Nature **359**(1992)393;

A. N. Taylor and M. Rowan-Robinson, Nature **359**(1992)396;

R. K. Schaefer and Q. Shafi, preprint BA-92-28,1992;

J. A. Holtzman and J. R. Primack, Astrophys. J. **405**(1993)428.

[8] J. R. Primack, J. Holtzman, A. Klypin and D.O.Caldwell, Phys. Rev. Lett. **74**(1995)2160.

[9] D. O. Caldwell and R. N. Mohapatra, Phys. Rev. **D48**(1993) 3259;

J. Peltoniemi and J. W. F. Valle, Nucl. Phys. **B406**(1993)409;

J. Peltoniemi, D. Tommasini and J. W. F. Valle, Phys. Lett. **B298**(1993) 383.

Z. G. Berezhiani and R. N. Mohapatra, Phys. Rev. **D52**(1995)6607.

D. O. Caldwell and R. N. Mohapatra, Phys. Lett. **B354**(1995)371.

[10] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity ed. P. van Nieuwenhuizen and D. Freedman (North-Holland,1979), p.315;

T. Yanagida, in proc. Workshop on Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK, 1979).

[11] A. Masiero, D. V. Nanopoulos and A. I. Sanda, Phys. Rev. Lett. **57**(1986)663;

B. A. Campbell, J. Ellis, K. Enquvist, M. K.Gaillard and D. V. Nanopoulos, Int. J. Mod. Phys. **A2**(1987)831;

K. S. Babu and R. N. Mohapatra, Phys. Rev. **D46**(1992)374.

[12] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive and H.-S. Kong, Astrophys. J. **376**(1991)51, and references therein.

[13] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**(1985) [Sov. J. Nucl. Phys. **42**(1985)913];

L. Wolfenstein, Phys. Rev. **D17**(1978)2369; Phys. Rev. **D20**(1979)2634.

[14] D. O. Caldwell and P. Langacker, Phys. Rev. **D44**(1991)823.

[15] X. Shi, D. N. Schramm and D. B. Fields, Phys. Rev. **D48**(1993)2563.

[16] D. Suematsu, Prog. Theor. Phys. **96**(1996)611.

[17] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **72**(1994)17.

[18] R. Barbieri and A. Dolgov, Phys. Lett. **B237**(1990)440; Nucl. Phys. **B349**(1991)743; K. Enqvist, K. Kainulainen and J. Maalampi, Phys. Lett. **B249**(1990)531; K. Kainulainen, Phys. Lett. **B244**(1990)191.

[19] R. Foot and R. R. Volkas, Phys. Rev. Lett, **75**(1995)4350.

[20] H. Fritzsch, Phys. Lett. **B70**(1977)436.

[21] D. Suematsu and Y. Yamagishi, Int. J. Mod. Phys. **A10**(1995)4521, and references therein.

[22] E. Nardi, Phys. Rev. **D48**(1992)3277.